A Quantum Macro Assembler

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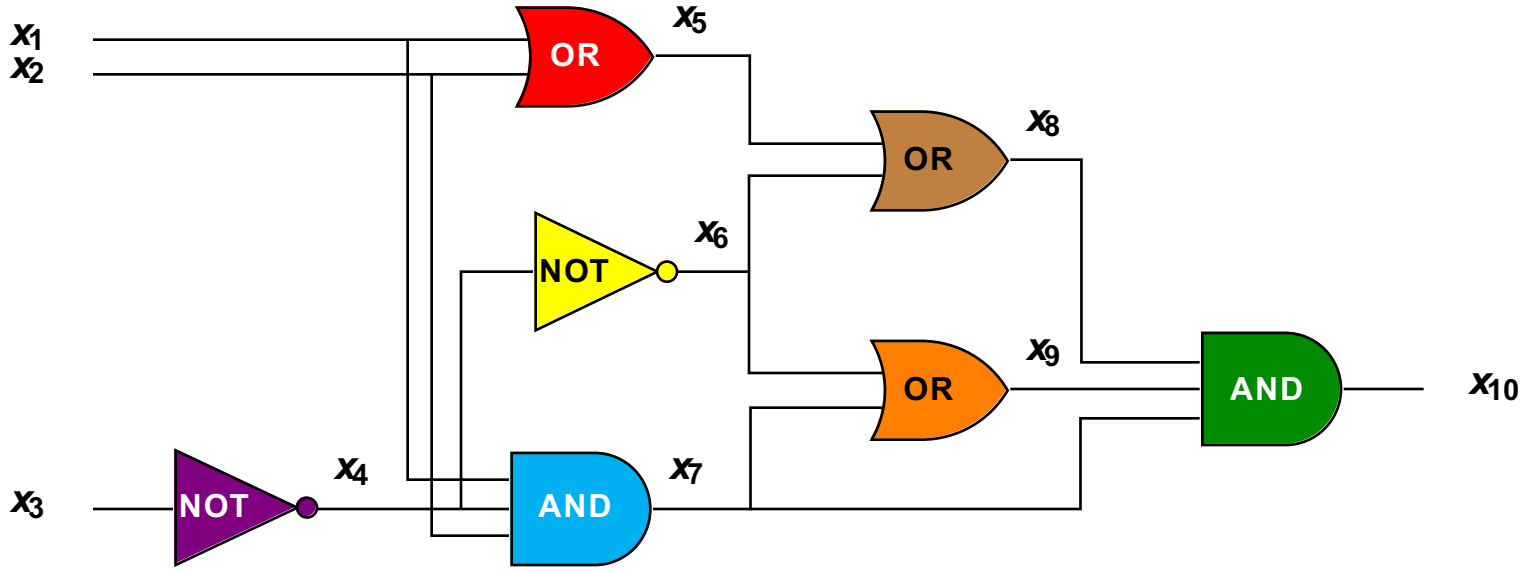
Outline

- Background and motivation
- Design and implementation
- Results and analysis
- Conclusions and future work

Motivating Example



- Circuit-satisfiability problem
- For what inputs (if any) is the output of a given circuit *true*?
- Classic NP-complete problem—can't beat exhaustive search in the general case (although usable heuristics do exist)

Refresher: Low-Level Programming Model

- **Ising-model Hamiltonian with at most two-spin interactions:**

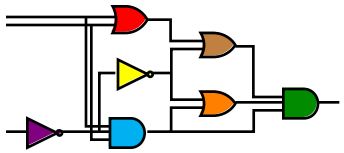
$$\arg\min_{\sigma} E(\sigma) = \arg\min_{\sigma} \left(\sum_{i=1}^{N} h_i \sigma_i + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} J_{i,j} \sigma_i \sigma_j \right)$$

Point (node) weights

Coupler (edge) strengths

for given $h \in \Re^N$, $J \in \Re^{N \times N}$ and solving for $\sigma \in \{-1, +1\}^N$

The Problem



- **The preceding expression does not lead to a particularly user-friendly way to write programs:**

```
0 0 -0.5
0 7 1
0 96 -1
2 6 -1
2 98 -1
6 14 -1
7 15 -1
7 7 -0.5
8 12 -1
8 13 -0.0909
8 14 0.04545
8 15 -1
8 8 -0.13635
9 105 0.3636
9 12 -0.0909
9 13 -1
9 14 -0.0909
9 9 0.1818
10 10 0.1818
10 106 -1
10 12 -0.1818
10 13 -0.1818
10 14 -0.1818
11 11 -0.13635
11 12 0.04545
11 13 -0.0909
11 14 -1
12 12 -0.13635
13 13 0.1818
14 14 -0.13635
15 23 -1
16 16 -0.5
16 22 1
16 23 -1
18 22 -1
22 22 -0.5
96 102 -1
96 192 -1
97 100 -0.6667
97 103 -1
97 97 -0.33335
98 100 0.3333
98 103 -0.6667
98 98 0.3333
100 100 0.3333
100 108 -1
102 110 -1
103 103 -0.33335
103 111 -1
105 108 -1
106 106 0.1818
106 108 -0.3636
106 202 -1
110 110 0.16665
110 118 -1
111 119 -1
112 112 -0.6667
112 118 -0.6667
112 119 -0.6667
112 208 -1
113 113 -0.5
113 117 0.25
113 209 -1
114 114 0.16665
114 118 0.3333
114 119 -1
117 117 -0.25
118 118 0.16665
119 119 0.16665
192 192 0.16665
192 196 0.16665
192 197 -1
192 198 -0.6667
194 194 0.16665
194 196 -1
194 197 0.16665
194 198 -0.6667
196 196 0.16665
196 204 -1
197 197 0.16665
198 198 -0.6667
198 206 -1
202 204 -1
204 212 -1
206 214 -1
208 215 -1
209 209 0.3636
209 212 -0.1818
209 213 -0.1818
209 214 -0.3636
209 215 -0.1818
210 210 0.1818
210 212 -0.0909
210 213 -1
210 214 0.3636
210 215 -0.1818
211 211 -0.13635
211 212 -1
211 213 -0.0909
211 215 0.0909
212 212 -0.13635
213 213 0.1818
215 215 -0.2727
```

Physical Representations Exposed Throughout

- **Must consider physical connectivity of the on-chip network**
  - Set of available couplers is very sparse: at most 6 $J_{i,j}$ for a given i
  - Any given installation will have a number of missing qubits (nodes) and couplers (edges) on the chip—differs from installation to installation
  - *Limitation*: No 3-cycles (or any odd cycles); must work around by converting to 4-cycles
  - Think manual place-and-route
- **Must consider physical range of coefficients**
  - Varies from installation to installation but typically around [-2, +2] for point weights and [-1, +1] for coupler strengths
- **Results are given in terms of physical qubit numbers**

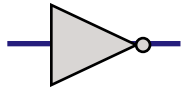
Goal

- **Abstract away physical system characteristics**
  - Still have to understand underlying D-Wave architecture and properties
- **Macros to support code reuse**
- **Analogy to classical assembly language**
  - On Intel 64, `imull` maps to `0000 1111 1010 1111` for $reg_1 \times reg_2$ but to `0110 1011` for $reg_1 \times imm$
  - More convenient to write (and remember) `imull`, and no expressiveness is lost

➔ **Implement a *quantum macro assembler***

- Use as a building block for (future) higher-level programming languages

A First Look at QASM



- **Let's define a NOT operator in terms of a Hamiltonian:**

σ_1	σ_2	$E(\sigma) = -\frac{1}{2}\sigma_1 + -\frac{1}{2}\sigma_2 + 1\sigma_1\sigma_2$
-1	-1	2
-1	+1	-1
+1	-1	-1
+1	+1	0

All the effort goes into computing appropriate *h* and *J* coefficients

By design, arg $\min_\sigma E(\sigma)$ occurs exactly where σ_2 = NOT σ_1

- **Here's the corresponding (and complete) QASM program for NOT:**

```
s1 -0.5
s2 -0.5
s1 s2 1.0
```

s1 and s2 are arbitrary variable names. (*I was going to use "bele" and "lokai" but figured that reference is too obscure.*)

Running a QASM Program

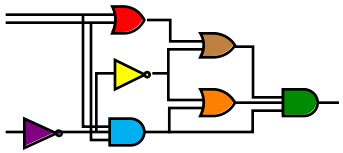
```
$ qasm --run not.qasm
# s1 --> 774
# s2 --> 782
Solution #1 (energy = -1.00):

    Name(s)   Spin   Boolean
    -------   ----   -------
    s1          -1   False
    s2          +1   True

Solution #2 (energy = -1.00):

    Name(s)   Spin   Boolean
    -------   ----   -------
    s1          +1   True
    s2          -1   False
```

Compiles and runs the program

Logical-to-physical mapping

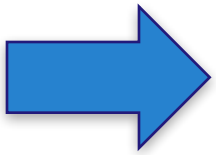
Solutions reported in terms of the symbolic names used in the source code

Taking Advantage of Macros



- **Our sample circuit contains *two* NOT gates**
- **Let's wrap up our definition in a macro to facilitate reuse:**

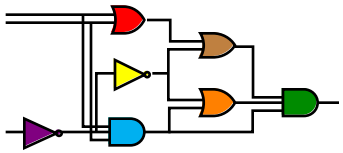
```
s1 -0.5
s2 -0.5
s1 s2 1.0
```



```
# NOT (Y = not A)

!begin_macro not1
$A -0.5
$Y -0.5
$A $Y 1.0
!end_macro not1
```

- **Any variable name containing "$" is considered "internal" or "uninteresting" and not output by default**
  - `--verbose --verbose` will show it, though
  - In our example, we don't care about individual operators, just the overall circuit inputs and outputs

A Circuit Library

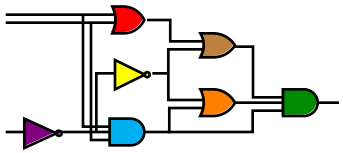


- **For our sample circuit, we need a 3-input AND and a 2-input OR in addition to the NOT we just defined**

```
# 3-input AND (Y = A and B and C)

!begin_macro and3
$A  -0.2727
$B   0.0000
$C  -0.2727
$Y   0.3636
$a1  0.3636

$A $B   0.0000
$A $C   0.0909
$A $Y  -0.1818
$A $a1 -0.1818
$B $C   0.0000
$B $Y  -0.3636
$B $a1  0.3636
$C $Y  -0.1818
$C $a1 -0.1818
$Y $a1 -0.1818
!end_macro and3
```

```
# 2-input OR (Y = A or B)

!begin_macro or2
$A  0.3333
$B  0.3333
$Y -0.6667

$A $B  0.3333
$A $Y -0.6667
$B $Y -0.6667
!end_macro or2
```

Wiring It Up



- **We use "=" to chain (equate) two variables**
  - Implementation: Choose $J_{i,j}<0$ to favor $\sigma_i = \sigma_j$

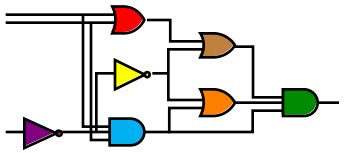
```
# Solve a circuit-satisfiability problem

!include <gates>

!use_macro not1 not_x4
not_x4.$A = x3
not_x4.$Y = $x4

!use_macro or2 or_x5
or_x5.$A = x1
or_x5.$B = x2
or_x5.$Y = $x5

!use_macro not1 not_x6
not_x6.$A = $x4
not_x6.$Y = $x6

!use_macro and3 and_x7
and_x7.$A = x1
and_x7.$B = x2
and_x7.$C = $x4
and_x7.$Y = $x7

!use_macro or2 or_x8
or_x8.$A = $x5
or_x8.$B = $x6
or_x8.$Y = $x8

!use_macro or2 or_x9
or_x9.$A = $x6
or_x9.$B = $x7
or_x9.$Y = $x9

!use_macro and3 and_x10
and_x10.$A = $x8
and_x10.$B = $x9
and_x10.$C = $x7
and_x10.$Y = x10
```

Aside: Why Not Use ToQ?

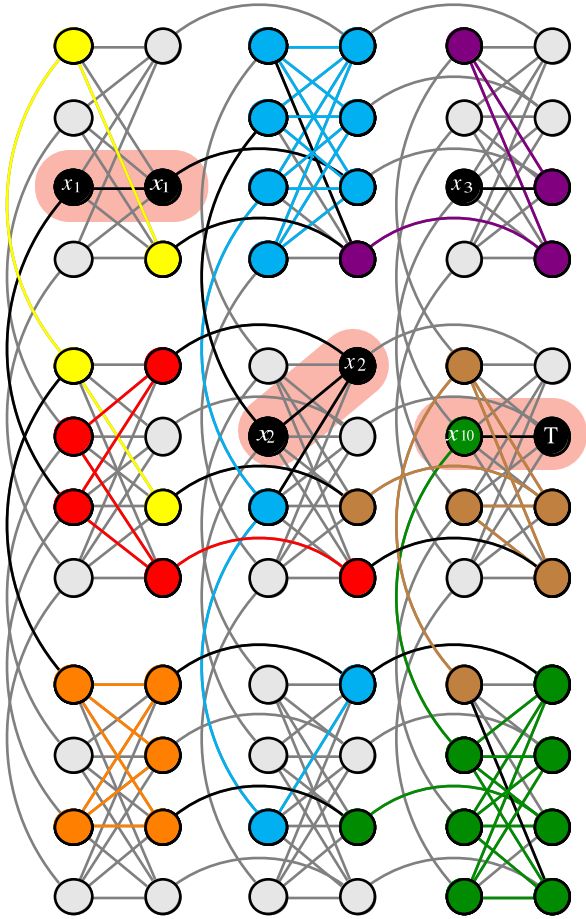
```
bool: @x1, @x2, @x3
bool: @x4, @x5, @x6
bool: @x7, @x8, @x9

assert: @x4 == Not(@x3)

assert: @x5 == Or(@x1, @x2)
assert: @x6 == Not(@x4)
assert: @x7 == And(And(@x1, @x2), @x4)

assert: @x8 == Or(@x5, @x6)
assert: @x9 == Or(@x6, @x7)

assert: And(And(@x8, @x9), @x7) == 1

end:
```

- **Higher-level programming model**
  - Constraint satisfaction
- **Pros**
  - Rich library of built-in functions (including Booleans)
  - User doesn't need to convert truth tables to Hamiltonians
- **Cons**
  - Substantial fraction of computation is performed classically in pre-processing step

Compilation

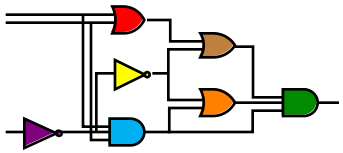


- **QASM uses D-Wave's heuristic embedder to map program variables to physical qubits**
  - Some variables are mapped to multiple qubits
- **Output (x_{10}) is pinned to *true* on the command line**
  - Technique: Use truth table in which both T=*false* → x_{10}=*true* and T=*true* → x_{10}=*true*



Logical feature	Tally	Physical feature	Tally
Variables	32	Qubits	53
Strengths	31	Couplers	71
Equivalences	21	Chains	23

Execution



```
$ qasm --pin="x10 := true" -O --run circsat.qasm
# x1 --> 204
# x10 --> 1
# x2 --> 194 197
# x3 --> 113
Solution #1 (energy = -39.00):

    Name(s)   Spin   Boolean
    -------   ----   -------
    x1          +1   True
    x10         +1   True
    x2          +1   True
    x3          -1   False
```

Performance

- **Measure both compilation time and execution time**
  - No clean distinction between program and inputs
- **Annealing time (time per solution) is a D-Wave 2X *input***
  - More samples per unit time vs. increased likelihood of finding ground state
  - For each of 100 runs, we took 1000 samples @ 20μs/sample

Code	Meaning	Time (s)	
A	Base compilation	0.98	Compilation
B	Optimization	1.01	
C	Device pre- and post-processing	0.27	Execution
D	Annealing	0.02	
E	Other (local prep. + HTTPS round trip)	1.47	
	Total	3.75	

Solution Quality

- **The D-Wave 2X is a stochastic device**
  - Can get different answers from one annealing cycle to the next
  - Can even get *wrong* answers
- **QASM can filter out obviously incorrect results**
  - Broken chains (where *x* should be equal to *y* but isn't)
  - Broken pins (where *x* should be *true* but is in fact *false* (or vice versa)
  - Non-ground state solutions (not arg $\min_{\sigma} E(\sigma)$)
- **QASM also ignores differences in "uninteresting" variables**

Solution type	Tally
All	1000
Distinct	169.2 ± 21.1
Valid	89.6 ± 8.6
Ground state	2.0 ± 0.1
Interesting	1.0 ± 0.0

(100 runs, 1000 samples/run)

Issues

- **Programmability**
  - Need fast algorithm for representing an arbitrary truth table as the ground state of a Hamiltonian
- **Performance**
  - Ought to cache embeddings locally and cache device state near the device to accelerate running same program with different inputs
  - Need faster embedder (possibly special-purpose); don't want to have to classically solve an NP-complete embedding problem before executing an NP-complete program on the D-Wave
- **Solution quality**
  - The D-Wave 2X seems sensitive to physical placement of qubits
  - Same basic circuit laid out differently on the Chimera graph can observe a radically different fraction of valid solutions

Conclusions

- **QASM provides an improvement in program expressibility and ease of use for quantum (really, any) annealers**
  - …in the sense that assembly language is easier to use than machine language
- **Low-level, but reduces the number of mundane details to worry about**
  - Embedding Hamiltonians in the D-Wave 2X's physical topology
  - Reasoning about the physical location(s) of each variable
  - Pinning Boolean values to variables
  - Reusing code blocks
  - Interpreting results
- **QASM can be a useful building block for higher-level programming models**
  - TBD what these might look like
  - *Current thinking*: sparsely connected islands of pre-computed logic blocks (not necessarily Boolean operators)

https://github.com/losalamos/qasm

(BSD-licensed but depends on D-Wave's proprietary APIs)